Constraining Neutron Star Mass-Radius Relation

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The TOV equations and the EoS

What are they, and why do we need them?
How to solve them?
What information they give us about our system (star) ?

The hydrostatic equilibrium equation

This equation can be derived from Newtonian gravitational theory, by combining the hydrostatic equation with the expression for g(r)

$$\frac{dP}{dr} = -\rho(r)g(r)$$

or,

$$\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2}$$

The equation for mass

This equation relates the mass to the density of a spherical object

$$\frac{d\mathcal{M}(r)}{dr} = 4\pi r^2 \rho(r)$$

The need of an EoS

In order to solve the equations we need another relation, a relation between pressure and density, this relation is called “equation of state”, and, unlike the TOVs, it depends upon the model used.

For white dwarfs we used the Fermi gas model for electrons.

density of states of electrons

$$dn = \frac{d^3k}{(2\pi\hbar)^3} = \frac{4\pi k^2\, dk}{(2\pi\hbar)^3}.$$

energy density of electrons

$$\epsilon_{elec}(k_F) = \frac{8\pi}{(2\pi\hbar)^3}\int_0^{k_F} (k^2c^2 + m_e^2c^4)^{1/2} k^2\, dk$$

The need of an EoS

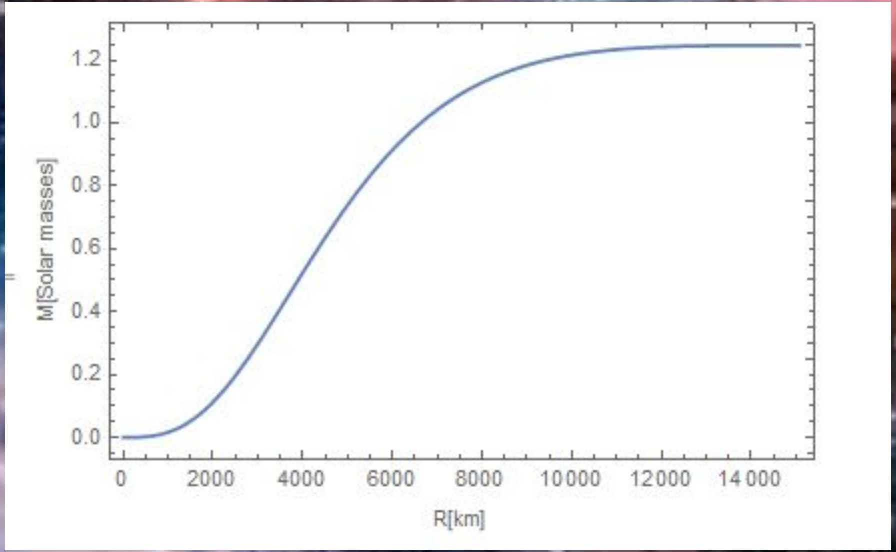
Once we calculate the energy of the electrons we just add the energy of the nucleons to get the total energy, using the first law of thermodynamics we can find the pressure, allowing us to find, in the end a relation between pressure and energy density.

$$\epsilon = n m_N A/Z + \epsilon_{elec}(k_F).$$

$$p = \left[-\frac{\partial U}{\partial V} \right]_{T=0} = n^2 \frac{d(\epsilon/n)}{dn} = n \frac{d\epsilon}{dn} - \epsilon = n\mu - \epsilon$$

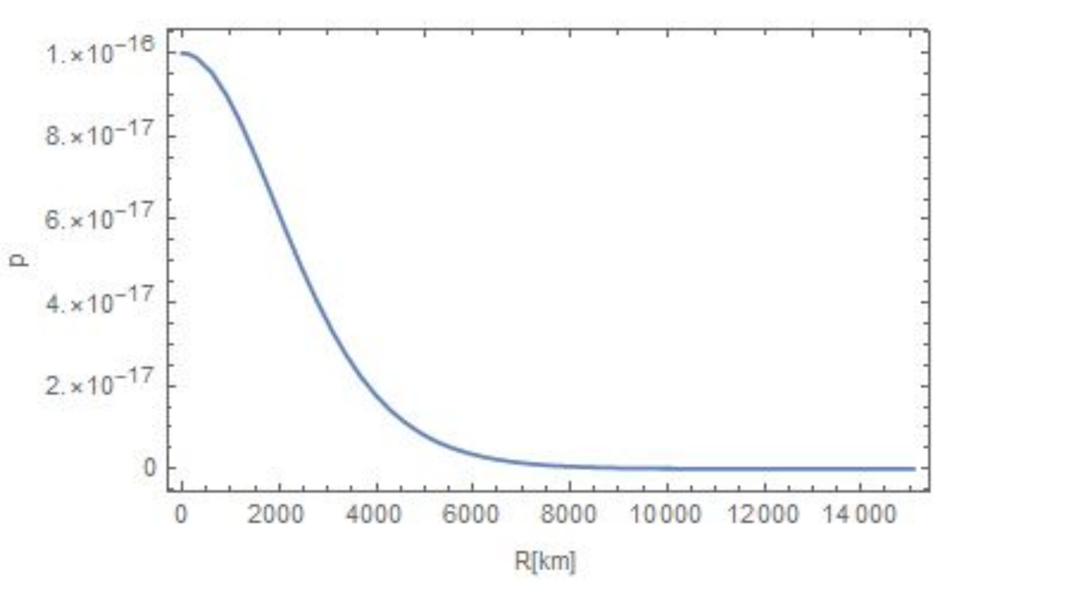
Calculation of the profile of a white dwarf

```
In[ ]:= sol = NDSolve[{p'[x] == -1.473*p[x]^(3/4) * m[x]/x^2, m'[x] == 52.46*x^2*p[x]^(3/4), m[0.01] == 0, p[0.01] == 10^(-16)}, {p, m}, {x, 0.01, 15080}]
```



Calculation of the profile of a white dwarf

Pressure profile

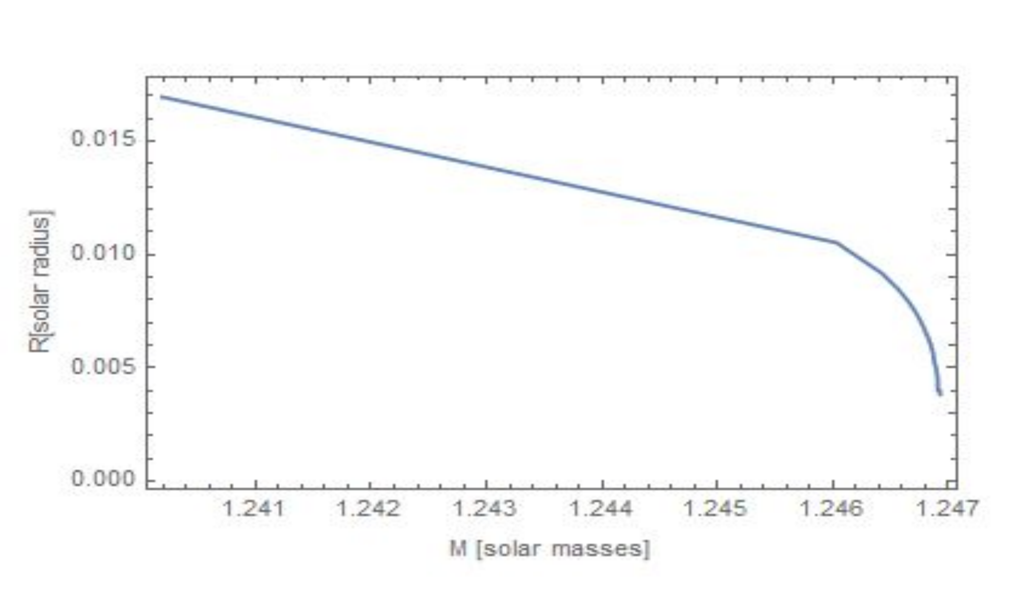


Mass radius relation for white dwarfs

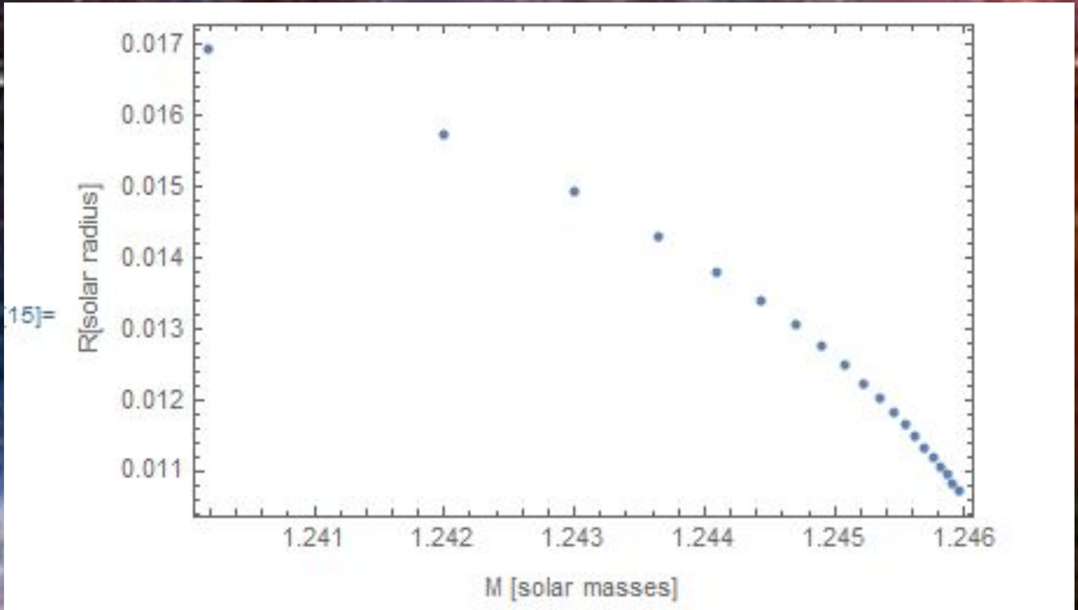
```
Pcmin = 10^(-16);
Pcmax = 10^(-14);
Np = 100;
dP = 10*(Pcmax - Pcmin)/(Np - 1);
MR = {};

For[i = 0, i < Np, i++,
 sol = NDSolve[{p'[x] == -1.473*p[x]^(3/4) * m[x]/x^2, m'[x] == 52.46*x^2*p[x]^(3/4) , m[0.01] == 0, p[0.01] == Pcmin + i*dP,
     WhenEvent[p[x] < 10^(-20), MR = Append[MR, {m[x], x/(6.9*10^5)}]]}, {p, m}, {x, 0.01, 11700}];


]
MR
```

Mass radius relation for white dwarfs

For a relativistic Fermi electron gas

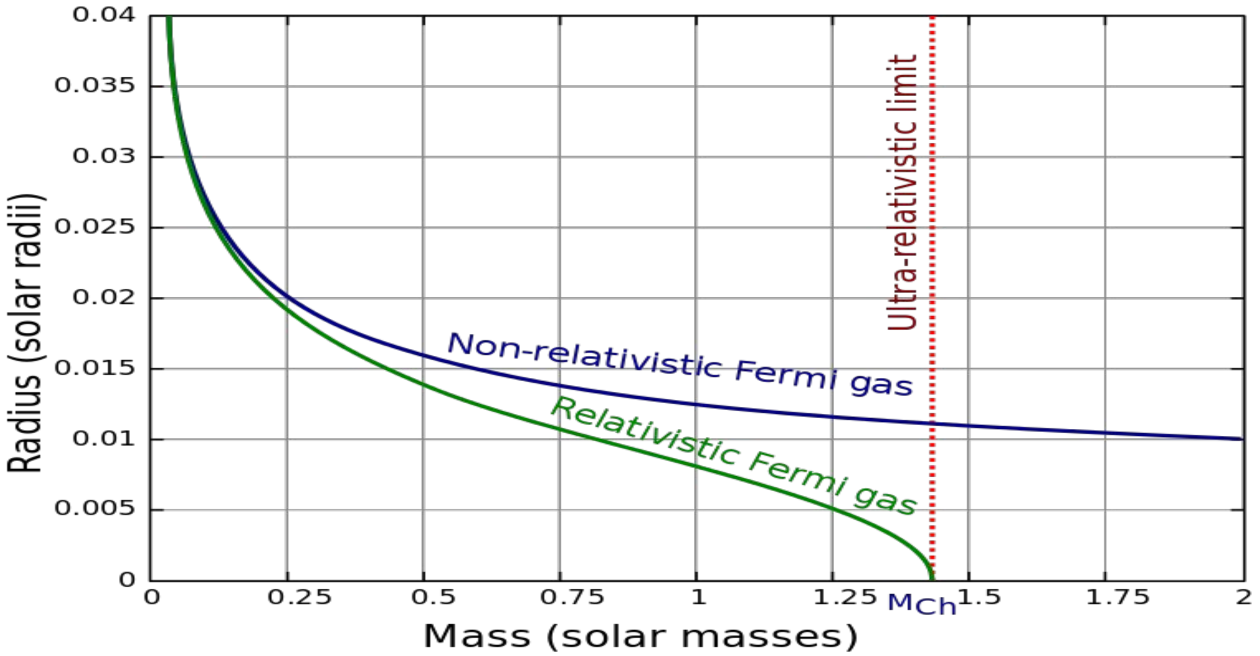


Mass radius relation for white dwarfs



Mass radius relation for white dwarfs

From wikipedia



Outlines

- Introduction of neutron star
- Different models of neutron star
- Tolman-Volkoff-Oppenheimer (TOV) equations
- Equation of state (EoS)
- Mass-Radius relations
- Conclusions

Introduction

- What is neutron star ?

- How do they form ?

- Why the study of neutron star is relevant ?

Different Models

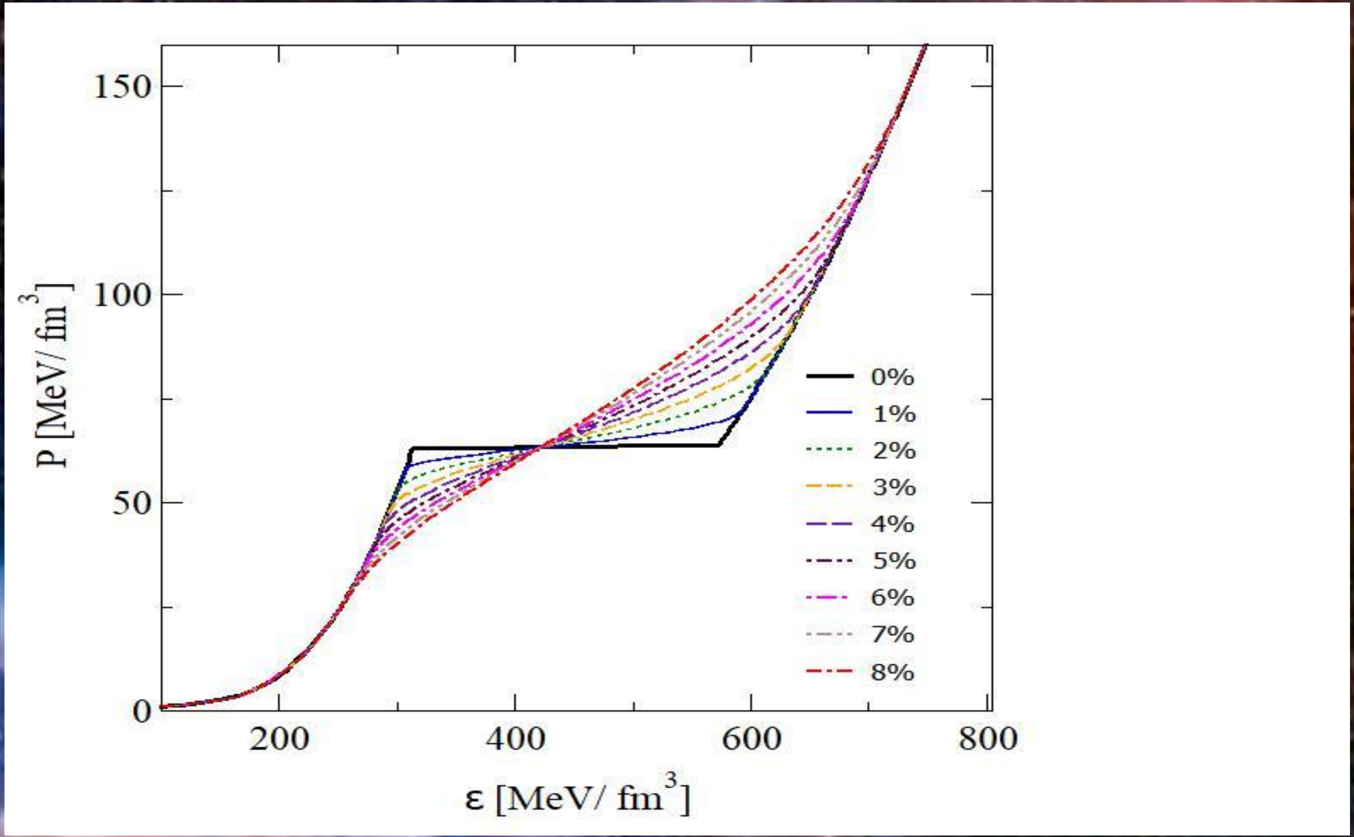
- Pure Neutron star
- Neutron star with protons and electrons
- Neutrons star with protons and electrons, nuclear interactions are also taken into account.

TOV equations

$$\frac{d\mathcal{M}(r)}{dr} = 4\pi r^2 \rho(r) = \frac{4\pi r^2 \epsilon(r)}{c^2}$$

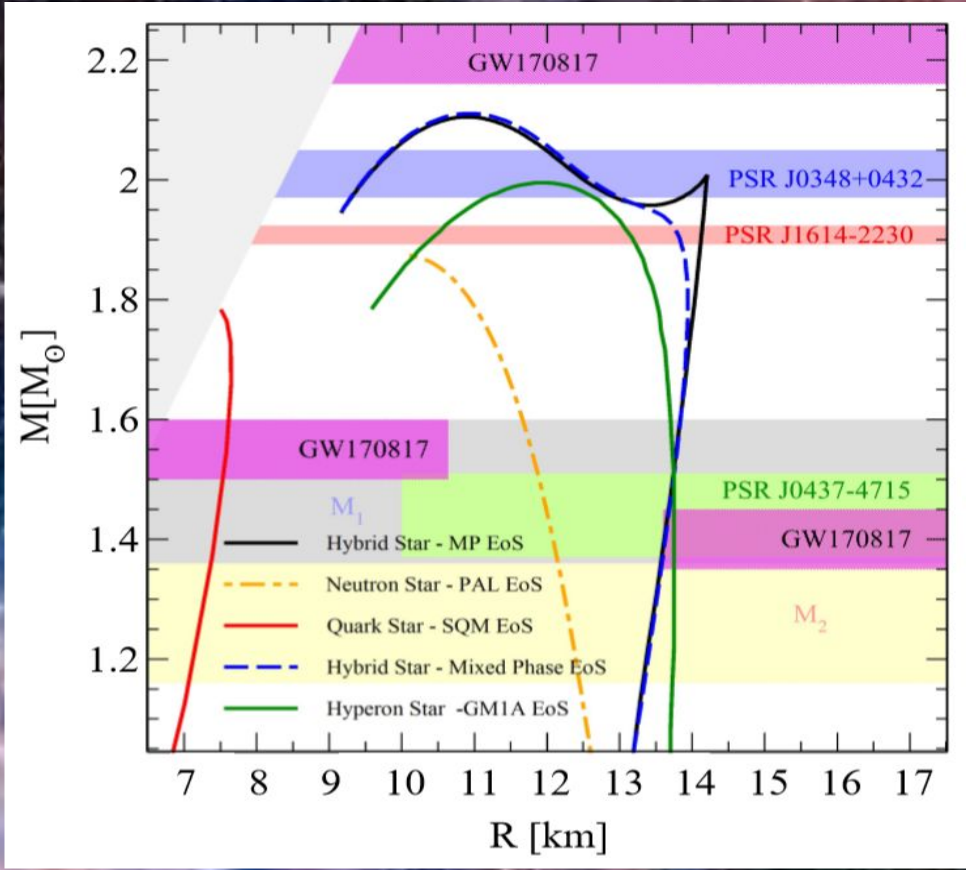
$$\frac{dp}{dr} = -\frac{G\epsilon(r)\mathcal{M}(r)}{c^2 r^2}\left[1 + \frac{p(r)}{\epsilon(r)}\right]\left[1 + \frac{4\pi r^3 p(r)}{\mathcal{M}(r)c^2}\right] \times \left[1 - \frac{2G\mathcal{M}(r)}{c^2 r}\right]^{-1}.$$

Equation of State



Source: From Dr. D. Alvarez

Mass-Radius Relations



Source : From Dr. D. Alvarez

Conclusions

- Study of mass-radius relation can exclude a model.

References

- Neutron stars for undergraduates, Richard R. Silbar and Sanjay Reddy.


Thank You !